FYJC - MATHEMATICS & STATISTICS

PAPER - I

COMPOUND ANGLES

COMPILED & CONDUCTED @ JKSC

COMPOUND ANGLES - 5.5

MULTIPLE & SUBMULTIPLE ANGLES

$\sin = 2 \sin . \cos$

$1 - \cos = 2\sin^2$

$1 + \cos = 2\cos^2$

$\tan = \dfrac{2 \tan}{1 - \tan^2}$

$\cos = \cos^2 - \sin^2$

$1 + \sin = (\cos^2 + \sin^2 + 2.\sin \cos)$
$= (\cos + \sin)^2$

$1 - \sin = (\cos^2 + \sin^2 - 2.\sin \cos)$
$= (\cos - \sin)^2$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Q SET - 1

01 $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$

02 . $\dfrac{1 - \cos 2x}{\sin 2x} = \tan x$

03. $\dfrac{\sin 2\theta}{1 - \cos 2\theta} = \cot \theta$

04. $\dfrac{\sin x}{1 - \cos x} = \cot x$

05. $\sqrt{\dfrac{1 - \cos 2x}{1 + \cos 2x}} = \tan x$

06. $\sqrt{\dfrac{1 + \cos A}{1 - \cos A}} = \cot A/2$

07. $\dfrac{1 + \cos A + \cos 2A}{\sin A + \sin 2A} = \cot A$

08. $\dfrac{1 + \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta} = \cot \theta/2$

Q SET - 2

01. $\sqrt{\dfrac{1 - \sin 2A}{1 + \sin 2A}} = \dfrac{1 - \tan A}{1 + \tan A}$

02. $\dfrac{\cos 2A}{1 - \sin 2A} = \dfrac{1 + \tan A}{1 - \tan A}$

03. $\dfrac{\cos\theta}{1 + \sin\theta} = \dfrac{1 - \tan(\theta/2)}{1 + \tan(\theta/2)}$

04. $\dfrac{\cos A}{1 + \sin A} = \dfrac{\cot(A/2) - 1}{\cot(A/2) + 1}$

05. $\sqrt{\dfrac{1 + \sin\theta}{1 - \sin\theta}} = \tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)$

06. $\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \frac{1}{\sec\theta - \tan\theta}$

Q SET - 3

01.
$\frac{\sin 16\theta}{\sin\theta} = 16\cos\theta \cdot \cos 2\theta \cdot \cos 4\theta \cdot \cos 8\theta$

02.
$\cos 5^\circ \cdot \cos 10^\circ \cdot \cos 20^\circ \cdot \cos 40^\circ = \frac{\cos 10^\circ}{16\ \sin 5^\circ}$

03.
$\cos 3^\circ \cdot \cos 6^\circ \cdot \cos 12^\circ \cdot \cos 24^\circ = \frac{\cos 42^\circ}{16\ \sin 3^\circ}$

04.
$\cos 7^\circ \cdot \cos 14^\circ \cdot \cos 28^\circ \cdot \cos 56^\circ = \frac{\sin 68^\circ}{16\cos 83^\circ}$

05. $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$

06. $\sqrt{2 + \sqrt{2 + 2\cos 2A}} = 2\cos {}^{A}/_{2}$

07. $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}} = 2\cos\theta$

08. $\frac{\sin 3A}{\cos A} + \frac{\cos 3A}{\sin A} = 2\cot 2A$

Q SET - 4

01
If $\sec\theta = \frac{-13}{5}$, ${}^{\pi}/_{2} < \theta < \pi$.
Find a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\tan 2\theta$

02
If $\sin\theta = -\frac{4}{5}$, $\pi < \theta < {}^{3\pi}/_{2}$.
Find a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\tan 2\theta$

03
If $\tan x = -\frac{3}{4}$, ${}^{3\pi}/_{2} < x < 2\pi$.
Find a) $\sin 2x$ b) $\cos 2x$ c) $\tan 2x$

Q SET - 5

01.
$\cos 10^\circ \cdot \cos 30^\circ \cdot \cos 50^\circ \cdot \cos 70^\circ = 3/16$

02.
$\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = 3/16$

03.
$4.\cos A \cdot \cos({}^{\pi}/_{3} - A) \cdot \cos({}^{\pi}/_{3} + A) = \cos 3A$

04.
$\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = 1/16$

05.
$\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = 3/16$

06.
$4.\sin A \cdot \sin({}^{\pi}/_{3} - A) \cdot \sin({}^{\pi}/_{3} + A) = \sin 3A$

07.
$\tan 20^\circ \cdot \tan 40^\circ \cdot \tan 60^\circ \cdot \tan 80^\circ = 3$

08. $x + \frac{1}{x} = 2\cos\theta$,
then prove $x^3 + \frac{1}{x^3} = 2\cos 3\theta$

Q SET - 6

01. $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$

02. $\frac{\sec 2A - 1}{\sec 4A - 1} = \frac{\tan A}{\tan 4A}$

03. $2\operatorname{cosec} 2x + \operatorname{cosec} x = \sec x \cdot \cot({}^{x}/_{2})$

04.
$\frac{2\cos 4x + 1}{2\cos x + 1} = (2\cos x - 1)(2\cos 2x - 1)$

SOLUTION TO Q SET - 1

01 $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$

LHS $= \dfrac{2 \sin x . \cos x}{2\cos^2 x}$

$= \dfrac{\sin x}{\cos x}$

$= \tan x = \text{RHS}$

02 . $\dfrac{1 - \cos 2x}{\sin 2x} = \tan x$

LHS $= \dfrac{2\sin^2 x}{2\sin x . \cos x}$

$= \dfrac{\sin x}{\cos x}$

$= \tan x = \text{RHS}$

03. $\dfrac{\sin 2\theta}{1 - \cos 2\theta} = \cot \theta$

LHS $= \dfrac{2\sin \theta . \cos \theta}{2\sin^2\theta}$

$= \dfrac{\cos \theta}{\sin \theta}$

$= \cot \theta = \text{RHS}$

04. $\dfrac{\sin x}{1 - \cos x} = \cot x$

LHS $= \dfrac{2\sin {}^x/_2 . \cos {}^x/_2}{2\sin^2 {}^x/_2}$

$= \dfrac{\cos {}^x/_2}{\sin {}^x/_2}$

$= \tan {}^x/_2 = \text{RHS}$

05. $\sqrt{\dfrac{1 - \cos 2x}{1 + \cos 2x}} = \tan x$

LHS $= \sqrt{\dfrac{2 \sin^2 x}{2\cos^2 x}}$

$= \dfrac{\sin x}{\cos x}$

$= \tan x = \text{RHS}$

06. $\sqrt{\dfrac{1 + \cos A}{1 - \cos A}} = \cot {}^A/_2$

LHS $= \sqrt{\dfrac{2 \cos^2 {}^A/_2}{2 \sin^2 {}^A/_2}}$

$= \dfrac{\cos {}^A/_2}{\sin {}^A/_2}$

$= \cot {}^A/_2 = \text{RHS}$

07. $\dfrac{1 + \cos A + \cos 2A}{\sin A + \sin 2A} = \cot A$

LHS $= \dfrac{1 + \cos 2A + \cos A}{\sin 2A + \sin A}$

$= \dfrac{2\cos^2 A + \cos A}{2\sin A . \cos A + \sin A}$

$= \dfrac{\cos A(2\cos A + 1)}{\sin A\ (2\cos A + 1)}$

$= \dfrac{\cos A}{\sin A}$

$= \cot A = \text{RHS}$

08. $\dfrac{1 + \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta} = \cot {}^{\theta}/_{2}$

$$\text{LHS} = \frac{1 + \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta}$$

$$= \frac{2\cos^2 {}^{\theta}/_{2} + 2\sin {}^{\theta}/_{2}.\cos {}^{\theta}/_{2}}{2\sin^2 {}^{\theta}/_{2} + 2\sin {}^{\theta}/_{2}.\cos {}^{\theta}/_{2}}$$

$$= \frac{2\cos {}^{\theta}/_{2} \; (\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2})}{2\sin {}^{\theta}/_{2} \; (\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2})}$$

$$= \frac{\cos {}^{\theta}/_{2}}{\sin {}^{\theta}/_{2}}$$

$$= \cot {}^{\theta}/_{2} \quad = \text{RHS}$$

SOLUTION TO Q SET - 2

01. $\sqrt{\dfrac{1 - \sin 2A}{1 + \sin 2A}} = \dfrac{1 - \tan A}{1 + \tan A}$

$$\text{LHS} = \sqrt{\frac{\cos^2 A + \sin^2 A - 2\sin A.\cos A}{\cos^2 A + \sin^2 A + 2\sin A.\cos A}}$$

$$= \sqrt{\frac{(\cos A - \sin A)^2}{(\cos A + \sin A)^2}}$$

$$= \frac{\cos A - \sin A}{\cos A + \sin A}$$

Dividing Numerator & Denominator by cos A

$$= \frac{\dfrac{\cos A - \sin A}{\cos A}}{\dfrac{\cos A + \sin A}{\cos A}}$$

$$= \frac{1 - \tan A}{1 + \tan A}$$

= RHS

02. $\dfrac{\cos 2A}{1 - \sin 2A} = \dfrac{1 + \tan A}{1 - \tan A}$

$$\text{LHS} = \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A - 2\sin A.\cos A}$$

$$= \frac{(\cos A - \sin A).(\cos A + \sin A)}{(\cos A - \sin A)^2}$$

$$= \frac{\cos A + \sin A}{\cos A - \sin A}$$

Dividing Numerator & Denominator by cos A

$$= \frac{\dfrac{\cos A + \sin A}{\cos A}}{\dfrac{\cos A - \sin A}{\cos A}}$$

$$= \frac{1 + \tan A}{1 - \tan A} \quad = \text{RHS}$$

03. $\dfrac{\cos\theta}{1 + \sin\theta} = \dfrac{1 - \tan(\theta/2)}{1 + \tan(\theta/2)}$

$$\text{LHS} = \frac{\cos^2 {}^{\theta}/_{2} - \sin^2 {}^{\theta}/_{2}}{\cos^2 {}^{\theta}/_{2} + \sin^2 {}^{\theta}/_{2} + 2\sin {}^{\theta}/_{2}.\cos {}^{\theta}/_{2}}$$

$$= \frac{(\cos {}^{\theta}/_{2} - \sin {}^{\theta}/_{2}).(\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2})}{(\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2})^2}$$

$$= \frac{\cos {}^{\theta}/_{2} - \sin {}^{\theta}/_{2}}{\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2}}$$

Dividing Numerator & Denominator by cos A

$$= \frac{\dfrac{\cos {}^{\theta}/_{2} - \sin {}^{\theta}/_{2}}{\cos {}^{\theta}/_{2}}}{\dfrac{\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2}}{\cos {}^{\theta}/_{2}}}$$

$$= \frac{1 - \tan {}^{\theta}/_{2}}{1 + \tan {}^{\theta}/_{2}} \quad = \text{RHS}$$

04. $\dfrac{\cos A}{1 + \sin A} = \dfrac{\cot (A/2) - 1}{\cot (A/2) + 1}$

$$\text{LHS} = \frac{\cos^2 A/2 - \sin^2 A/2}{\cos^2 A/2 + \sin^2 A/2 + 2\sin A/2 . \cos A/2}$$

$$= \frac{(\cos A/2 - \sin A/2) . (\cos A/2 + \sin A/2)}{(\cos A/2 + \sin A/2)^2}$$

$$= \frac{\cos A/2 - \sin A/2}{\cos A/2 + \sin A/2}$$

Dividing Numerator & Denominator by $\sin A/2$

$$= \frac{\dfrac{\cos A/2 - \sin A/2}{\sin A/2}}{\dfrac{\cos A/2 + \sin A/2}{\sin A/2}}$$

$$= \frac{\cot A/2 - 1}{\cot A/2 + 1} = \text{RHS}$$

05. $\sqrt{\dfrac{1 + \sin \theta}{1 - \sin \theta}} = \tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)$

$$\text{LHS} = \sqrt{\frac{\cos^2 \theta/2 + \sin^2 \theta/2 + 2\sin \theta/2 . \cos \theta/2}{\cos^2 \theta/2 + \sin^2 \theta/2 - 2\sin \theta/2 . \cos \theta/2}}$$

$$= \sqrt{\frac{(\cos \theta/2 + \sin \theta/2)^2}{(\cos \theta/2 - \sin \theta/2)^2}}$$

$$= \frac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2 - \sin \theta/2}$$

Dividing Numerator & Denominator by $\cos \theta/2$

$$= \frac{\dfrac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2}}{\dfrac{\cos \theta/2 - \sin \theta/2}{\cos \theta/2}}$$

$$= \frac{1 + \tan \theta/2}{1 - \tan \theta/2} = \text{RHS}$$

$$\text{RHS} = \tan (\pi/4 + \theta/2)$$

$$= \frac{\tan \pi/4 + \tan \theta/2}{1 - \tan \pi/4 \tan \theta/2}$$

$$= \frac{1 + \tan \theta/2}{1 - \tan \theta/2}$$

LHS = RHS

06. $\tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right) = \dfrac{1}{\sec \theta - \tan \theta}$

$$\text{RHS} = \frac{1}{\sec \theta - \tan \theta}$$

$$= \frac{1}{\dfrac{1}{\cos \theta} - \dfrac{\sin \theta}{\cos \theta}}$$

$$= \frac{\cos \theta}{1 - \sin\theta}$$

$$= \frac{\cos^2 \theta/2 - \sin^2 \theta/2}{\cos^2 \theta/2 + \sin^2 \theta/2 - 2\sin \theta/2 . \cos \theta/2}$$

$$= \frac{(\cos \theta/2 - \sin \theta/2) . (\cos \theta/2 + \sin \theta/2)}{(\cos \theta/2 - \sin \theta/2)^2}$$

$$= \frac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2 - \sin \theta/2}$$

Dividing Numerator & Denominator by cos A

$$= \frac{\dfrac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2}}{\dfrac{\cos \theta/2 - \sin \theta/2}{\cos \theta/2}}$$

$$= \frac{1 + \tan \theta/2}{1 - \tan \theta/2}$$

RHS $= \tan\left(\pi/4 + \theta/2\right)$

$= \dfrac{\tan \pi/4 + \tan \theta/2}{1 - \tan \pi/4 \tan \theta/2}$

$= \dfrac{1 + \tan \theta/2}{1 - \tan \theta/2}$

LHS = RHS

SOLUTION TO Q SET - 3

01.

$\sin 16\theta = 16 \cos\theta \,.\, \cos 2\theta \,.\, \cos 4\theta \,.\, \cos 8\theta$

$\sin\theta$

We Prove

$16 \sin\theta \,.\, \cos\theta \,.\, \cos 2\theta \,.\, \cos 4\theta \,.\, \cos 8\theta = \sin 16\theta$

LHS

$= 2.2.2.(2 \sin\theta \cos\theta).\, \cos 2\theta \,.\, \cos 4\theta \,.\, \cos 8\theta$

$= 2.2.(2.\sin 2\theta.\, \cos 2\theta) \,.\, \cos 4\theta \,.\, \cos 8\theta$

$= 2.(2.\sin 4\theta \,.\, \cos 4\theta) \,.\, \cos 8\theta$

$= 2.\sin 8\theta \,.\, \cos 8\theta$

$= \sin 16\theta$ = RHS

02.

$\cos 5^\circ \,.\, \cos 10^\circ \,.\, \cos 20^\circ \,.\, \cos 40^\circ = \dfrac{\cos 10^\circ}{16 \sin 5^\circ}$

We Prove

$16 \sin 5^\circ \,.\, \cos 5^\circ \,.\, \cos 10^\circ \,.\, \cos 20^\circ \,.\, \cos 40^\circ = \cos 10^\circ$

LHS

$= 16 \sin 5^\circ.\cos 5^\circ.\, \cos 10^\circ \,.\, \cos 20^\circ \,.\, \cos 40^\circ$

$= 2.2.2.\, \underline{2\sin 5^\circ.\cos 5^\circ}\, \cos 10^\circ.\cos 20^\circ.\cos 40^\circ$

$= 2.2.\, \underline{2.\sin 10^\circ \cos 10^\circ}\, \cos 20^\circ \,.\, \cos 40^\circ$

$= 2.\, \underline{2 \sin 20^\circ \cos 20^\circ}\, \cos 40^\circ$

$= 2 \sin 40^\circ \cos 40^\circ$

$= \sin 80^\circ$

$= \sin(90^\circ - 80^\circ)$

$= \cos 10^\circ$

03.

$\cos 3^\circ \,.\, \cos 6^\circ \,.\, \cos 12^\circ \,.\, \cos 24^\circ = \dfrac{\cos 42^\circ}{16 \sin 3^\circ}$

We Prove

$16 \sin 3^\circ \,.\, \cos 3^\circ \,.\, \cos 6^\circ \,.\, \cos 12^\circ \,.\, \cos 24^\circ = \cos 42^\circ$

LHS

$= 16 \sin 3^\circ.\cos 3^\circ.\, \cos 6^\circ \,.\, \cos 12^\circ \,.\, \cos 24^\circ$

$= 2.2.2.\, \underline{2\sin 3^\circ.\cos 3^\circ}\, \cos 6^\circ.\cos 12^\circ.\, \cos 24^\circ$

$= 2.2.\, \underline{2.\sin 6^\circ \cos 6^\circ}\, \cos 12^\circ \,.\, \cos 24^\circ$

$= 2.\, \underline{2 \sin 12^\circ \cos 12^\circ}\, \cos 24^\circ$

$= 2 \sin 24^\circ \cos 24^\circ$

$= \sin 48^\circ$

$= \sin(90^\circ - 42^\circ)$

$= \cos 42^\circ$

04.

$\cos 7^\circ \,.\, \cos 14^\circ \,.\, \cos 28^\circ \,.\, \cos 56^\circ = \dfrac{\sin 68^\circ}{16\cos 83^\circ}$

We Prove

$16\cos 83^\circ \,.\, \cos 7^\circ \,.\, \cos 14^\circ \,.\, \cos 28^\circ \,.\, \cos 56^\circ = \sin 68^\circ$

LHS

$= 16\cos 83^\circ.\cos 7^\circ.\, \cos 14^\circ \,.\, \cos 18^\circ \,.\, \cos 56^\circ$

$= 16 \sin 7^\circ.\cos 7^\circ.\, \cos 14^\circ \,.\, \cos 18^\circ \,.\, \cos 56^\circ$

$= 2.2.2.\underline{2\sin 7^\circ.\cos 7^\circ}\ \cos 14^\circ.\cos 28^\circ.\ \cos 56^\circ$

$= 2.2.\ \underline{2.\sin 14^\circ \cos 14^\circ}\ \cos 28^\circ\ .\ \cos 56^\circ$

$= 2.\ \underline{2\sin 28^\circ \cos 28^\circ}\ \ \cos 56^\circ$

$= 2\sin 56^\circ\ \cos 56^\circ$

$= \sin 112^\circ$

$= \sin(180^\circ - 68^\circ)$

$= \sin 68^\circ$

05.

$$\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$$

LHS

$= \sqrt{2 + \sqrt{2(1 + \cos 4\theta)}}$

$= \sqrt{2 + \sqrt{2\ .\ 2\cos^2 2\theta}}$

$= \sqrt{2 + \sqrt{4\cos^2 2\theta}}$

$= \sqrt{2 + 2\cos 2\theta}$

$= \sqrt{2(1 + \cos 2\theta)}$

$= \sqrt{2\ .\ 2\cos^2\theta}$

$= \sqrt{4\cos^2\theta}$

$= 2\cos\theta$

$=$ RHS

06.

$$\sqrt{2 + \sqrt{2 + 2\cos 2A}} = 2\cos A/2$$

LHS

$= \sqrt{2 + \sqrt{2(1 + \cos 2A)}}$

$= \sqrt{2 + \sqrt{2\ .\ 2\cos^2 A}}$

$= \sqrt{2 + \sqrt{4\cos^2 A}}$

$= \sqrt{2 + 2\cos A}$

$= \sqrt{2(1 + \cos A)}$

$= \sqrt{2\ .\ 2\cos^2 A/2}$

$= \sqrt{4\cos^2 A/2}$

$= 2\cos A/2$

$=$ RHS

07.

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}} = 2\cos\theta$$

LHS

$= \sqrt{2 + \sqrt{2 + \sqrt{2(1 + \cos 8\theta)}}}$

$= \sqrt{2 + \sqrt{2 + \sqrt{2\ .\ 2\cos^2 4\theta}}}$

$= \sqrt{2 + \sqrt{2 + \sqrt{4\cos^2 4\theta}}}$

$= \sqrt{2 + \sqrt{2 + 2\cos 4\theta}}$

$= \sqrt{2 + \sqrt{2(1 + \cos 4\theta)}}$

$= \sqrt{2 + \sqrt{2 \cdot 2\cos^2 2\theta}}$

$= \sqrt{2 + \sqrt{4\cos^2 2\theta}}$

$= \sqrt{2 + 2\cos 2\theta}$

$= \sqrt{2(1 + \cos 2\theta)}$

$= \sqrt{2 \cdot 2\cos^2\theta}$

$= \sqrt{4\cos^2\theta}$

$= 2\cos\theta$

08. $\dfrac{\sin 3A}{\cos A} + \dfrac{\cos 3A}{\sin A} = 2\cot 2A$

LHS

$= \dfrac{\sin 3A \cdot \sin A + \cos 3A \cdot \cos A}{\sin A \cdot \cos A}$

$= \dfrac{\cos 3A \cdot \cos A + \sin 3A \cdot \sin A}{\sin A \cdot \cos A}$

$= \dfrac{\cos(3A - A)}{\sin A \cdot \cos A}$

$= \dfrac{\cos 2A}{\sin A \cdot \cos A}$

$= \dfrac{2\cos 2A}{2\sin A \cdot \cos A}$

$= \dfrac{2\cos 2A}{\sin 2A}$

$= 2\cot 2A = \text{RHS}$

SOLUTION TO Q SET - 4

01

If $\sec\theta = \frac{-13}{5}$, $\pi/2 < \theta < \pi$.

Find a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\tan 2\theta$

SOLUTION

θ lies in II Quadrant . $\sin\theta$ & $\operatorname{cosec}\theta$ are +

$$\cos\theta = -\frac{5}{13}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \frac{25}{169} = 1$$

$$\sin^2\theta = 1 - \frac{25}{169}$$

$$\sin^2\theta = \frac{144}{169}$$

$$\sin\theta = +\frac{12}{13}$$

a) $\sin 2\theta = 2\sin\theta \, . \cos\theta$

$$= 2\ \frac{12}{13}\ \frac{-5}{13}$$

$$= -\frac{120}{169}$$

b) $\cos 2\theta = 1 - 2\sin^2\theta$

$$= 1 - 2\ \frac{144}{169}$$

$$= \frac{169 - 288}{169}$$

$$= -\frac{119}{169}$$

c) $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

$$= \frac{120}{119}$$

02

If $\sin\theta = -\frac{4}{5}$, $\pi < \theta < 3\pi/2$.

Find a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\tan 2\theta$

SOLUTION

θ lies in III Quadrant . $\tan\theta$ & $\cot\theta$ are +

$$\sin\theta = -\frac{4}{5}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta + \frac{16}{25} = 1$$

$$\cos^2\theta = \frac{9}{25}$$

$$\cos\theta = -\frac{3}{5}$$

a) $\sin 2\theta = 2\sin\theta \, . \cos\theta$

$$= 2\ \frac{-4}{5}\ \frac{-3}{5}$$

$$= \frac{24}{25}$$

b) $\cos 2\theta = 1 - 2\sin^2\theta$

$$= 1 - 2\ \frac{16}{25}$$

$$= \frac{25 - 32}{25}$$

$$= -\frac{7}{25}$$

c) $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

$$= -\frac{24}{7}$$

03

If $\tan x = -\frac{3}{4}$, $\frac{3\pi}{2} < x < 2\pi$.

Find a) $\sin 2x$ b) $\cos 2x$ c) $\tan 2x$

SOLUTION

x lies in IV Quadrant . cos x & sec x are +

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \frac{9}{16} = \sec^2 x$$

$$\sec^2 x = \frac{25}{16}$$

$$\sec x = +\frac{5}{4}$$

$$\cos x = \frac{4}{5}$$

$$\cos^2 x + \sin^2 x = 1$$

$$\frac{16}{25} + \sin^2 x = 1$$

$$\sin^2 x = \frac{9}{25}$$

$$\sin x = -\frac{3}{5}$$

a) $\sin 2x = 2 \sin x . \cos x$

$$= 2 \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$$

$$= -\frac{24}{25}$$

b) $\cos 2x = 1 - 2\sin^2 x$

$$= 1 - 2\left(\frac{9}{25}\right)$$

$$= \frac{25 - 18}{25}$$

$$= \frac{7}{25}$$

c) $\tan 2x = \frac{\sin 2x}{\cos 2x}$

$$= -\frac{24}{7}$$

SOLUTION TO Q SET - 5

01. $\cos 10^\circ . \cos 30^\circ . \cos 50^\circ . \cos 70^\circ = 3/16$

LHS $= \cos 10^\circ . \cos 30^\circ . \cos 50^\circ . \cos 70^\circ$

$$= \frac{\sqrt{3}}{2} . \cos 10^\circ . \cos (60 - 10)^\circ . \cos (60 + 10)^\circ$$

$$= \frac{\sqrt{3}}{2} . \cos 10^\circ . (\cos 60 \cos 10 + \sin 60 \sin 10) . (\cos 60 \cos 10 - \sin 60 \sin 10)$$

$$= \frac{\sqrt{3}}{2} . \cos 10^\circ . \left(\frac{1}{2}\cos 10 + \frac{\sqrt{3}}{2}\sin 10\right) . \left(\frac{1}{2}\cos 10 - \frac{\sqrt{3}}{2}\sin 10\right)$$

$$= \frac{\sqrt{3}}{2} . \cos 10^\circ . \left(\frac{1}{4}\cos^2 10 - \frac{3}{4}\sin^2 10\right)$$

$$= \frac{\sqrt{3}}{8} . \cos 10^\circ . \left(\cos^2 10 - 3 \sin^2 10\right)$$

$$= \frac{\sqrt{3}}{8} . \cos 10^\circ . \left(\cos^2 10 - 3 (1 - \cos^2 10)\right)$$

$$= \frac{\sqrt{3}}{8} . \cos 10^\circ . \left(\cos^2 10 - 3 + 3\cos^2 10\right)$$

$$= \frac{\sqrt{3}}{8} . \cos 10^\circ . (4\cos^2 10 - 3)$$

$$= \frac{\sqrt{3}}{8} . (4\cos^3 10 - 3 \cos 10^\circ)$$

$4\cos^3\theta - 3\cos \theta = \cos 3\theta$

$$= \frac{\sqrt{3}}{8} . \cos 30^\circ$$

$$= \frac{\sqrt{3}}{8} . \frac{\sqrt{3}}{2}$$

$$= \frac{3}{16}$$

02. $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = 3/16$

LHS $= \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ$

$= \frac{1}{2} \cdot \cos 20^\circ \cdot \cos (60 - 20)^\circ \cdot \cos (60 + 20)^\circ$

$= \frac{1}{2} \cdot \cos 20^\circ \cdot (\cos 60 \cos 20 + \sin 60 \sin 20) \cdot (\cos 60 \cos 20 - \sin 60 \sin 20)$

$= \frac{1}{2} \cdot \cos 20^\circ \cdot \left(\frac{1}{2} \cos 20 + \frac{\sqrt{3}}{2} \sin 20\right) \cdot \left(\frac{1}{2} \cos 20 - \frac{\sqrt{3}}{2} \sin 20\right)$

$= \frac{1}{2} \cdot \cos 20^\circ \cdot \left(\frac{1}{4} \cos^2 20 - \frac{3}{4} \sin^2 20\right)$

$= \frac{1}{8} \cdot \cos 20^\circ \cdot \left(\cos^2 20 - 3 \sin^2 20\right)$

$= \frac{1}{8} \cdot \cos 20^\circ \cdot \left(\cos^2 20 - 3 (1 - \cos^2 20)\right)$

$= \frac{1}{8} \cdot \cos 20^\circ \cdot \left(\cos^2 20 - 3 + 3\cos^2 20\right)$

$= \frac{1}{8} \cdot \cos 20^\circ \cdot (4\cos^2 20 - 3)$

$= \frac{1}{8} \cdot (4\cos^3 20 - 3 \cos 20^\circ)$

$4\cos^3\theta - 3\cos \theta = \cos 3\theta$

$= \frac{1}{8} \cdot \cos (2 \times 20^\circ)$

$= \frac{1}{8} \cdot \cos 60^\circ$

$= \frac{1}{8} \cdot \frac{1}{2}$

$= \frac{1}{16}$

03. $4.\cos A \,.\, \cos(^{\pi}/_3 - A) \,.\, \cos(^{\pi}/_3 + A) = \cos 3A$

LHS $= 4.\cos A \,.\, \cos(60 - A) \,.\, \cos(60 + A)$

$= 4.\cos A \,.\, (\cos 60 \cos A + \sin 60 \sin A) \,.\, (\cos 60 \cos A - \sin 60 \sin A)$

$= 4 \,.\, \cos A \,.\, \left(\frac{1}{2}\cos A + \frac{\sqrt{3}}{2}\sin A\right) . \left(\frac{1}{2}\cos A - \frac{\sqrt{3}}{2}\sin A\right)$

$= 4.\cos A \,.\, \left(\frac{1}{4}\cos^2 A - \frac{3}{4}\sin^2 A\right)$

$= \cos A \,.\, \left[\cos^2 A - 3(1 - \cos^2 A)\right]$

$= \cos A \,.\, \left[4\cos^2 A - 3\right]$

$= 4\cos^3 A - 3\cos A \qquad = \cos 3A$

04. $\sin 10^\circ \,.\, \sin 30^\circ \,.\, \sin 50^\circ \,.\, \sin 70^\circ = 1/16$

LHS $= \sin 10^\circ \,.\, \sin 30^\circ \,.\, \sin 50^\circ \,.\, \sin 70^\circ$

$= \frac{1}{2} . \sin 10^\circ \,.\, \sin(60 - 10)^\circ \,.\, \sin(60 + 10)^\circ$

$= \frac{1}{2} . \sin 10^\circ \,.\, (\sin 60 \cos 10 - \cos 60 \sin 10) \,.\, (\sin 60 \cos 10 + \cos 60 \sin 10)$

$= \frac{1}{2} . \sin 10^\circ \,.\, \left(\frac{\sqrt{3}}{2}\cos 10 + \frac{1}{2}\sin 10\right) . \left(\frac{\sqrt{3}}{2}\cos 10 + \frac{1}{2}\sin 10\right)$

$= \frac{1}{2} . \sin 10^\circ \,.\, \left(\frac{3}{4}\cos^2 10 - \frac{1}{4}\sin^2 10\right)$

$= \frac{1}{8} . \sin 10^\circ \,.\, \left[3\cos^2 10 - \sin^2 10\right]$

$= \frac{1}{8} . \sin 10^\circ \,.\, \left[3(1 - \sin^2 10) - \sin^2 10\right]$

$= \frac{1}{8} . \sin 10^\circ \,.\, \left[3 - 3\sin^2 10 - \sin^2 10\right]$

$= \frac{1}{8} . \sin 10^\circ \,.\, (3 - 4\sin^2 10)$

$= \frac{1}{8} . (3\sin 10^\circ - 4\sin^3 10)$

$= \frac{1}{8} . \sin(3 \times 10^\circ)$

$= \frac{1}{8} . \sin 30$

$= \frac{1}{8} . \frac{1}{2}$

$= \frac{1}{16}$

05. $\sin 20^\circ . \sin 40^\circ . \sin 60^\circ . \sin 80^\circ = 3/16$

$$\text{LHS} = \sin 20^\circ . \sin 40^\circ . \sin 60^\circ . \sin 80^\circ$$

$$= \frac{\sqrt{3}}{2} . \sin 20^\circ . \sin (60 - 20)^\circ . \sin (60 + 20)^\circ$$

$$= \frac{\sqrt{3}}{2} . \sin 20^\circ . (\sin 60 \cos 20 - \cos 60 \sin 20) . (\sin 60 \cos 20 + \cos 60 \sin 20)$$

$$= \frac{\sqrt{3}}{2} . \sin 20^\circ . \left(\frac{\sqrt{3}}{2}\cos 20 + \frac{1}{2}\sin 20\right) . \left(\frac{\sqrt{3}}{2}\cos 20 + \frac{1}{2}\sin 20\right)$$

$$= \frac{\sqrt{3}}{2} . \sin 20^\circ . \left(\frac{3}{4}\cos^2 20 - \frac{1}{4}\sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \sin 20^\circ . \left(3 \cos^2 20 - \sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \sin 20^\circ . \left(3 (1 - \sin^2 20) - \sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \sin 20^\circ . \left(3 - 3 \sin^2 20 - \sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \sin 20^\circ . (3 - 4\sin^2 20)$$

$$= \frac{\sqrt{3}}{8} . (3 \sin 20^\circ - 4 \sin^3 20)$$

$3\sin \theta - 4\sin^3\theta = \sin 3\theta$

$$= \frac{\sqrt{3}}{8} . \sin (3 \times 20^\circ)$$

$$= \frac{\sqrt{3}}{8} . \sin 30^\circ$$

$$= \frac{\sqrt{3}}{8} . \frac{1}{2}$$

$$= \frac{3}{16}$$

06. $4.\sin A \cdot \sin\left(\frac{\pi}{3} - A\right) \cdot \sin\left(\frac{\pi}{3} + A\right) = \sin 3A$

LHS $= 4.\sin A \cdot \sin(60 - A) \cdot \sin(60 + A)$

$= 4.\sin A \cdot (\sin 60 \cos A - \cos 60 \sin A) \cdot (\sin 60 \cos A + \cos 60 \sin A)$

$= 4 \cdot \sin A \cdot \left(\frac{\sqrt{3}}{2}\cos A - \frac{1}{2}\sin A\right) \cdot \left(\frac{\sqrt{3}}{2}\cos A + \frac{1}{2}\sin A\right)$

$= 4.\sin A \cdot \left(\frac{3}{4}\cos^2 A - \frac{1}{4}\sin^2 A\right)$

$= \sin A \cdot \left[3(1 - \sin^2 A) - \sin^2 A\right]$

$= \sin A \cdot \left[3 - 4\sin^2 A\right]$

$= 3\sin A - 4\sin^3 A$

$= \sin 3A$

07. $\tan 20^\circ \cdot \tan 40^\circ \cdot \tan 60^\circ \cdot \tan 80^\circ = 3$

LHS $= \tan 20^\circ \cdot \tan 40^\circ \cdot \tan 60^\circ \cdot \tan 80^\circ$

$= \sqrt{3}\tan 20^\circ \cdot \tan(60 - 20)^\circ \cdot \tan(60 + 20)^\circ$

$= \sqrt{3}\tan 20^\circ \cdot \left(\frac{\tan 60 - \tan 20}{1 + \tan 60 \cdot \tan 20}\right) \cdot \left(\frac{\tan 60 + \tan 20}{1 - \tan 60 \cdot \tan 20}\right)$

$= \sqrt{3}\tan 20^\circ \cdot \left(\frac{\sqrt{3} - \tan 20}{1 + \sqrt{3} \cdot \tan 20}\right)\left(\frac{\sqrt{3} + \tan 20}{1 - \sqrt{3} \cdot \tan 20}\right)$

$= \sqrt{3}\tan 20^\circ \cdot \left(\frac{3 - \tan^2 20}{1 - 3\tan^2 20}\right)$

$= \sqrt{3}\left(\frac{3\tan 20^\circ - \tan^3 20^\circ}{1 - 3\tan^2 20^\circ}\right)$

$$\boxed{\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \tan 3\theta}$$

$= \sqrt{3}\ \tan(3 \times 20)^\circ$

$= \sqrt{3}\ \tan 60^\circ$

$= \sqrt{3} \cdot \sqrt{3}$

$= 3 \qquad = \text{RHS}$

08. $x + \dfrac{1}{x} = 2\cos\theta$, then prove $x^3 + \dfrac{1}{x^3} = 2\cos 3\theta$

$$\left(x + \frac{1}{x}\right)^3 = 8\cos^3\theta$$

$$x^3 + \frac{1}{x^3} + 3x.\frac{1}{x}\left(x + \frac{1}{x}\right) = 8\cos^3\theta$$

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8\cos^3\theta$$

$$x^3 + \frac{1}{x^3} + 3\ (2\cos\theta) = 8\cos^3\theta$$

$$x^3 + \frac{1}{x^3} + 6\cos\theta = 8\cos^3\theta$$

$$x^3 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta$$

$$x^3 + \frac{1}{x^3} = 2(4\cos^3\theta - 3\cos\theta)$$

$$x^3 + \frac{1}{x^3} = 2\cos 3\theta$$

SOLUTION TO Q SET - 6

01. $\dfrac{\sec 8A - 1}{\sec 4A - 1} = \dfrac{\tan 8A}{\tan 2A}$

$$\text{LHS} = \frac{\sec 8A - 1}{\sec 4A - 1}$$

$$= \frac{\dfrac{1}{\cos 8A} - 1}{\dfrac{1}{\cos 4A} - 1}$$

$$= \frac{\dfrac{1 - \cos 8A}{\cos 8A}}{\dfrac{1 - \cos 4A}{\cos 4A}}$$

$$= \frac{1 - \cos 8A}{\cos 8A} \times \frac{\cos 4A}{1 - \cos 4A}$$

$$= \frac{2\sin^2 4A}{\cos 8A} \times \frac{\cos 4A}{2\sin^2 2A}$$

$$= \frac{2.\sin 4A.\cos 4A}{\cos 8A} . \frac{\sin 4A}{2\sin^2 2A}$$

$$= \frac{\sin 8A}{\cos 8A} . \frac{2.\sin 2A.\cos 2A}{2\sin^2 2A}$$

$$= \frac{\sin 8A}{\cos 8A} . \frac{\cos 2A}{\sin 2A}$$

$$= \tan 8A . \cot 2A$$

$$= \frac{\tan 8A}{\tan 2A}$$

02. $\dfrac{\sec 2A - 1}{\sec 4A - 1} = \dfrac{\tan A}{\tan 4A}$

LHS = $\dfrac{\sec 2A - 1}{\sec 4A - 1}$

$= \dfrac{\dfrac{1}{\cos 2A} - 1}{\dfrac{1}{\cos 4A} - 1}$

$= \dfrac{\dfrac{1 - \cos 2A}{\cos 2A}}{\dfrac{1 - \cos 4A}{\cos 4A}}$

$= \dfrac{1 - \cos 2A}{\cos 2A} \times \dfrac{\cos 4A}{1 - \cos 4A}$

$= \dfrac{2\sin^2 A}{\cos 2A} \times \dfrac{\cos 4A}{2\sin^2 2A}$

$= \dfrac{2\sin^2 A}{\sin 2A} \times \dfrac{\cos 4A}{2\cos 2A.\sin 2A}$

$= \dfrac{2\sin^2 A}{2\sin A.\cos A} \times \dfrac{\cos 4A}{\sin 4A}$

$= \dfrac{\sin A}{\cos A} \times \dfrac{\cos 4A}{\sin 4A}$

$= \tan A \times \cot 4A$

$= \dfrac{\tan A}{\tan 4A}$

03. $2\operatorname{cosec} 2x + \operatorname{cosec} x = \sec x \,.\, \cot (x/2)$

LHS $= 2\operatorname{cosec} 2x + \operatorname{cosec} x$

$= \dfrac{2}{\sin 2x} + \dfrac{1}{\sin x}$

$= \dfrac{2}{2\sin x.\cos x} + \dfrac{1}{\sin x}$

$= \dfrac{1}{\sin x \,.\, \cos x} + \dfrac{1}{\sin x}$

$= \dfrac{1}{\sin x}\left(\dfrac{1}{\cos x} + 1\right)$

$= \dfrac{1}{\sin x} \quad \dfrac{1 + \cos x}{\cos x}$

$= \dfrac{1}{\cos x} \quad \dfrac{1 + \cos x}{\sin x}$

$= \sec x \quad \dfrac{2\cos^2(x/2)}{2\sin(x/2) .\cos (x/2)}$

$= \sec x \,.\, \dfrac{\cos(x/2)}{\sin(x/2)}$

$= \sec x \,.\, \cot(x/2)$

04. $\dfrac{2\cos 4x + 1}{2\cos x + 1} = (2\cos x - 1)\,(2\cos 2x - 1)$

NOTE

$\cos = \cos^2 - \sin^2$

$= \cos^2 - (1 - \cos^2)$

$= \cos^2 - 1 + \cos^2$

$= 2\cos^2 - 1$

LHS $= \dfrac{2\cos 4x + 1}{2\cos x + 1}$

$\cos = 2\cos^2 - 1$

$= \dfrac{2(2\cos^2 2x - 1) + 1}{2\cos x + 1}$

$= \dfrac{4.\cos^2 2x - 2 + 1}{2\cos x + 1}$

$= \dfrac{4.\cos^2 2x - 1}{2\cos x + 1}$

$= \dfrac{(2\cos 2x - 1)\,(2\cos 2x + 1)}{2\cos x + 1}$

$\cos = 2\cos^2 - 1$

$= \dfrac{(2\cos 2x - 1)\left[2\,(2\cos^2 x - 1) + 1\right]}{2\cos x + 1}$

$= \dfrac{(2\cos 2x - 1)\left[4\cos^2 x - 2 + 1\right]}{2\cos x + 1}$

$= \dfrac{(2\cos 2x - 1)\left[4\cos^2 x - 1\right]}{2\cos x + 1}$

$= \dfrac{(2\cos 2x - 1)(2\cos x - 1)\,(2\cos x + 1)}{2\cos x + 1}$

$= (2\cos 2x - 1)(2\cos x - 1)$

$=$ RHS

COMPOUND ANGLES - 5.6

FACTORISE & DEFACTORISE

FACTORIZATION FORMULAE

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$$

DE FACTORIZATION FORMULAE

$$2.\sin A . \cos B = \sin(A+B) + \sin(A-B)$$

$$2.\cos A . \sin B = \sin(A+B) - \sin(A-B)$$

$$2.\cos A . \cos B = \cos(A+B) + \cos(A-B)$$

$$2.\sin A . \sin B = \cos(A-B) - \cos(A+B)$$

Q SET - 1

01. $\dfrac{\cos 4\theta + \cos 2\theta}{\sin 4\theta - \sin 2\theta} = \cot\theta$

02. $\dfrac{\sin 5A + \sin 3A}{\cos 5A + \cos 3A} = \tan 4A$

03. $\dfrac{\sin 10\theta - \sin 2\theta}{\cos 2\theta - \cos 10\theta} = \cot 6\theta$

04. $\dfrac{\sin 8x + \sin 2x}{\cos 2x - \cos 8x} = \cot 3x$

05. $\dfrac{\cos 3\theta - \cos 11\theta}{\sin 11\theta - \sin 3\theta} = \tan 7\theta$

06. $\dfrac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha - \sin 2\beta} = \dfrac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}$

07. $\dfrac{\sin 2\alpha + \sin 2\beta}{\cos 2\alpha - \cos 2\beta} = \cot(\beta-\alpha)$

08. $\dfrac{\cos(7x-5y) + \cos(7y-5x)}{\sin(7x-5y) + \sin(7y-5x)} = \cot(x+y)$

Q SET - 2

01. $\dfrac{\sin A + \sin 4A + \sin 7A}{\cos A + \cos 4A + \cos 7A} = \tan 4A$

02. $\dfrac{\sin A + \sin 5A + \sin 9A}{\cos A + \cos 5A + \cos 9A} = \tan 5A$

03. $\dfrac{\sin 2\alpha + \sin 5\alpha - \sin\alpha}{\cos 2\alpha + \cos 5\alpha + \cos\alpha} = \tan 2\alpha$

04. $\dfrac{\sin 2\theta + 2.\sin 4\theta + \sin 6\theta}{\sin\theta + 2.\sin 3\theta + \sin 5\theta} = \cos\theta + \sin\theta . \cot 3\theta$

05. $\dfrac{\sin 3\theta + 2.\sin 5\theta + \sin 7\theta}{\sin \theta + 2.\sin 3\theta + \sin 5\theta} = \cos 2\theta + \sin 2\theta . \cot 3\theta$

06. $\dfrac{\cos 3A - 2.\cos 5A + \cos 7A}{\cos A - 2.\cos 3A + \cos 5A} = \cos 2A - \sin 2A . \tan 3A$

07. $\dfrac{\sin 5x - 2 \sin 3x + \sin x}{\cos 5x - \cos x} = \operatorname{cosec} 2x - \cot 2x$

Q SET - 3

01.

$\dfrac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A - \cos 3A - \cos 4A} = \cot A$

02.

$\dfrac{\sin x - \sin 3x + \sin 5x - \sin 7x}{\cos x - \cos 3x - \cos 5x + \cos 7x} = \cot 2x$

03.

$\dfrac{\sin x - \sin 5x + \sin 9x - \sin 13x}{\cos x - \cos 5x - \cos 9x + \cos 13x} = \cot 4x$

04.

if $\sin 2x + \sin 6x = \cos 2x + \cos 6x$

Show :

either $\tan 4x = 1$ OR $\cos 2x = 0$

05.

$\sin A + \sin 2A + \sin 3A = \cos A + \cos 2A + \cos 3A$

Show :

either $\tan 2A = 1$ OR $\cos A = -1/2$

06.

$\sin 10^\circ + \sin 50^\circ - \sin 80^\circ + \sin 140^\circ = \sqrt{2} . \sin 25^\circ$

07. $\cos 40^\circ + \cos 50^\circ + \cos 70^\circ + \cos 80^\circ$

$= \cos 20^\circ + \cos 10^\circ$

Q SET - 4

01.

$\dfrac{\sin 3A . \cos 4A - \sin A . \cos 2A}{\sin A . \sin 4A + \cos A . \cos 6A} = \tan 2A$

02.

$\dfrac{\sin 8\theta.\cos \theta - \sin 6\theta.\cos 3\theta}{\cos 2\theta.\cos \theta - \cos 3\theta.\cos 4\theta} = \cot 5\theta$

03.

$\dfrac{\sin 3\theta.\cos 5\theta - \sin \theta. \cos 7\theta}{\sin \theta. \sin 7\theta - \cos 3\theta.\cos 5\theta} = \tan 2\theta$

04.

$\dfrac{\cos 3A . \sin 9A - \sin A . \cos 5A}{\cos A . \cos 5A - \sin 3A . \sin 9A} = \tan 8A$

05.

$\dfrac{\cos 7A}{\cos A} = 2.\cos 6A - 2.\cos 4A + 2.\cos 2A - 1$

06.

$\sin\left(\dfrac{3A}{2}\right) . \sin\left(\dfrac{11A}{2}\right) + \sin\left(\dfrac{A}{2}\right) . \sin\left(\dfrac{7A}{2}\right)$

$= \sin 2A . \sin 5A$

Q SET - 5

01.

$\cos 20^\circ . \cos 100^\circ + \cos 100^\circ . \cos 140^\circ$

$- \cos 140^\circ . \cos 200^\circ = -3/4$

02.

$\cos 55^\circ . \cos 65^\circ + \cos 65^\circ . \cos 175^\circ +$

$\cos 175^\circ . \cos 55^\circ = -3/4$

03.

$\sin 25^\circ . \sin 35^\circ - \sin 25^\circ . \sin 85^\circ -$

$\sin 35^\circ . \sin 85^\circ = -3/4$

04.

$\cos^2 x + \cos^2(x + 120) + \cos^2(x - 120) = 3/2$

04.

$\cos^2 x + \cos^2(x + 300) + \cos^2(x - 300) = 3/2$

05.

$\sin^2\theta + \sin^2(120 + \theta) + \sin^2(120 - \theta) = 3/2$

SOLUTION TO Q SET - 1

01. $\dfrac{\cos 4\theta + \cos 2\theta}{\sin 4\theta - \sin 2\theta} = \cot\theta$

LHS

$$= \frac{2\cos\left(\frac{4\theta + 2\theta}{2}\right).\cos\left(\frac{4\theta - 2\theta}{2}\right)}{2\cos\left(\frac{4\theta + 2\theta}{2}\right).\sin\left(\frac{4\theta - 2\theta}{2}\right)}$$

$$= \frac{\cancel{2\cos 3\theta}.\cos\theta}{\cancel{2\cos 3\theta}.\sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

02. $\dfrac{\sin 5A + \sin 3A}{\cos 5A + \cos 3A} = \tan 4A$

LHS

$$= \frac{2\sin\left(\frac{5A + 3A}{2}\right).\cos\left(\frac{5A - 3A}{2}\right)}{2\cos\left(\frac{5A + 3A}{2}\right).\cos\left(\frac{5A - 3A}{2}\right)}$$

$$= \frac{2\sin 4A.\cancel{\cos A}}{2\cos 4A.\cancel{\cos A}}$$

$$= \frac{\sin 4A}{\cos 4A}$$

$$= \tan 4A$$

03. $\dfrac{\sin 10\theta - \sin 2\theta}{\cos 2\theta - \cos 10\theta} = \cot 6\theta$

LHS

$$= \frac{2\cos\left(\frac{10\theta + 2\theta}{2}\right)\sin\left(\frac{10\theta - 2\theta}{2}\right)}{-2\sin\left(\frac{2\theta + 10\theta}{2}\right).\sin\left(\frac{2\theta - 10\theta}{2}\right)}$$

$$= \frac{2\cos 6\theta.\sin 4\theta}{-2\sin 6\theta.\sin(-4\theta)}$$

$$= \frac{2\cos 6\theta.\cancel{\sin 4\theta}}{2\sin 6\theta.\cancel{\sin 4\theta}}$$

$$= \frac{\cos 6\theta}{\sin 6\theta}$$

$$= \cot 6\theta$$

04. $\dfrac{\sin 8x + \sin 2x}{\cos 2x - \cos 8x} = \cot 3x$

LHS

$$= \frac{2\sin\left(\frac{8x + 2x}{2}\right).\cos\left(\frac{8x - 2x}{2}\right)}{-2\sin\left(\frac{2x + 8x}{2}\right).\sin\left(\frac{2x - 8x}{2}\right)}$$

$$= \frac{2\sin 5x.\cos 3x}{-2\sin 5x.\sin(-3x)}$$

$$= \frac{\cancel{2\sin 5x}.\cos 3x}{\cancel{2\sin 5x}.\sin 3x}$$

$$= \frac{\cos 3x}{\sin 3x}$$

$$= \cot 3x$$

05. $\dfrac{\cos 3\theta - \cos 11\theta}{\sin 11\theta - \sin 3\theta} = \tan 7\theta$

LHS

$$= \frac{2\sin\left(\frac{3\theta + 11\theta}{2}\right)\sin\left(\frac{3\theta - 11\theta}{2}\right)}{-2\cos\left(\frac{11\theta + 3\theta}{2}\right).\sin\left(\frac{11\theta - 3\theta}{2}\right)}$$

$$= \frac{-2\sin 7\theta.\sin 4\theta}{-2\sin 7\theta.\sin 4\theta}$$

$$= \frac{2\sin 7\theta \, . \cancel{\sin 4\theta}}{2\cos 7\theta \, . \cancel{\sin 4\theta}}$$

$$= \frac{\sin 7\theta}{\cos 7\theta}$$

$$= \tan 7\theta$$

06. $\dfrac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha - \sin 2\beta} = \dfrac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}$

LHS

$$= \frac{2\sin\left(\frac{2\alpha+2\beta}{2}\right).\cos\left(\frac{2\alpha-2\beta}{2}\right)}{2\cos\left(\frac{2\alpha+2\beta}{2}\right).\sin\left(\frac{2\alpha-2\beta}{2}\right)}$$

$$= \frac{2\sin(\alpha+\beta) \, . \cos(\alpha-\beta)}{2\cos(\alpha+\beta) \, . \sin(\alpha-\beta)}$$

$$= \tan(\alpha+\beta) \, . \cot(\alpha-\beta)$$

$$= \frac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}$$

07. $\dfrac{\sin 2\alpha + \sin 2\beta}{\cos 2\alpha - \cos 2\beta} = \cot(\beta-\alpha)$

LHS

$$= \frac{2\sin\left(\frac{2\alpha+2\beta}{2}\right).\cos\left(\frac{2\alpha-2\beta}{2}\right)}{-2\sin\left(\frac{2\alpha+2\beta}{2}\right).\sin\left(\frac{2\alpha-2\beta}{2}\right)}$$

$$= \frac{2\sin(\alpha+\beta).\cos(\alpha-\beta)}{-2\sin(\alpha+\beta) \, . \sin(\alpha-\beta)}$$

$$= -\cot(\alpha-\beta)$$

$$= \cot(\beta-\alpha)$$

08. $\dfrac{\cos(7x-5y)+\cos(7y-5x)}{\sin(7x-5y)+\sin(7y-5x)} = \cot(x+y)$

$$= \frac{2\cos\left(\frac{7x-5y+7y-5x}{2}\right).\cos\left(\frac{7x-5y-7y+5x}{2}\right)}{2\sin\left(\frac{7x-5y+7y-5x}{2}\right).\cos\left(\frac{7x-5y-7y+5x}{2}\right)}$$

$$= \frac{2\cos\left(\frac{2x+2y}{2}\right)\cos\left(\frac{12x-12y}{2}\right)}{2\sin\left(\frac{2x+2y}{2}\right)\cos\left(\frac{12x-12y}{2}\right)}$$

$$= \frac{2\cos(x+y) \, . \cos(x-y)}{2\sin(x+y) \, .\cos(x-y)}$$

$$= \frac{\cos(x+y)}{\sin(x+y)}$$

$$= \cot(x+y)$$

SOLUTION TO Q SET - 2

01. $\dfrac{\sin A + \sin 4A + \sin 7A}{\cos A + \cos 4A + \cos 7A} = \tan 4A$

LHS

$$= \frac{\sin 7A + \sin A + \sin 4A}{\cos 7A + \cos A + \cos 4A}$$

$$= \frac{2\sin\left(\frac{7A+A}{2}\right).\cos\left(\frac{7A-A}{2}\right) + \sin 4A}{2\cos\left(\frac{7A+A}{2}\right).\cos\left(\frac{7A-A}{2}\right) + \cos 4A}$$

$$= \frac{2\sin 4A \, . \cos 3A + \sin 4A}{2\cos 4A \, .\cos 3A + \cos 4A}$$

$$= \frac{\sin 4A\cancel{(2\cos 3A + 1)}}{\cos 4A\cancel{(2\cos 3A + 1)}}$$

$$= \tan 4A$$

02. $\dfrac{\sin A + \sin 5A + \sin 9A}{\cos A + \cos 5A + \cos 9A} = \tan 5A$

LHS

$= \dfrac{\sin 9A + \sin A + \sin 5A}{\cos 9A + \cos A + \cos 5A}$

$= \dfrac{2\sin\left[\dfrac{9A+A}{2}\right].\cos\left[\dfrac{9A-A}{2}\right] + \sin 5A}{2\cos\left[\dfrac{9A+A}{2}\right].\cos\left[\dfrac{9A-A}{2}\right] + \cos 5A}$

$= \dfrac{2\sin 5A . \cos 4A + \sin 5A}{2\cos 5A . \cos 4A + \cos 5A}$

$= \dfrac{\sin 5A\,\cancel{(2\cos 4A + 1)}}{\cos 5A\cancel{(2\cos 4A + 1)}}$

$= \tan 5A$

03. $\dfrac{\sin 2\alpha + \sin 5\alpha - \sin\alpha}{\cos 2\alpha + \cos 5\alpha + \cos\alpha} = \tan 2\alpha$

LHS

$= \dfrac{\sin 2\alpha + \sin 5\alpha - \sin\alpha}{\cos 2\alpha + \cos 5\alpha + \cos\alpha}$

$= \dfrac{\sin 2\alpha + 2\cos\left[\dfrac{5\alpha+\alpha}{2}\right].\sin\left[\dfrac{5\alpha-\alpha}{2}\right]}{\cos 2\alpha + 2\cos\left[\dfrac{5\alpha+\alpha}{2}\right].\cos\left[\dfrac{5\alpha-\alpha}{2}\right]}$

$= \dfrac{\sin 2\alpha + 2\cos 4\alpha . \sin 2\alpha}{\cos 2\alpha + 2\cos 4\alpha . \cos 2\alpha}$

$= \dfrac{\sin 2\alpha\,\cancel{(1 + 2\cos 4\alpha)}}{\cos 2\alpha\cancel{(1 + 2\cos 4\alpha)}}$

$= \dfrac{\sin 2\alpha}{\cos 2\alpha}$

$= \tan 2\alpha$

04. $\dfrac{\sin 2\theta + 2.\sin 4\theta + \sin 6\theta}{\sin\theta + 2.\sin 3\theta + \sin 5\theta} = \cos\theta + \sin\theta . \cot 3\theta$

LHS

$= \dfrac{\sin 6\theta + \sin 2\theta + 2.\sin 4\theta}{\sin 5\theta + \sin\theta + 2.\sin 3\theta}$

$= \dfrac{2\sin\left[\dfrac{6\theta+2\theta}{2}\right].\cos\left[\dfrac{6\theta-2\theta}{2}\right] + 2.\sin 4\theta}{2\sin\left[\dfrac{5\theta+\theta}{2}\right].\cos\left[\dfrac{5\theta-\theta}{2}\right] + 2.\sin 3\theta}$

$= \dfrac{2\sin 4\theta . \cos 2\theta + 2.\sin 4\theta}{2\sin 3\theta . \cos 2\theta + 2.\sin 3\theta}$

$= \dfrac{2\sin 4\theta . \cancel{(\cos 2\theta + 1)}}{2\sin 3\theta . \cancel{(\cos 2\theta + 1)}}$

$= \dfrac{\sin 4\theta}{\sin 3\theta}$

$= \dfrac{\sin(3\theta + \theta)}{\sin 3\theta}$

$= \dfrac{\sin 3\theta . \cos\theta}{\sin 3\theta} + \dfrac{\cos 3\theta . \sin\theta}{\sin 3\theta}$

$= \cos\theta + \sin\theta . \cot 3\theta$

05. $\dfrac{\sin 3\theta + 2.\sin 5\theta + \sin 7\theta}{\sin\theta + 2.\sin 3\theta + \sin 5\theta} = \cos 2\theta + \sin 2\theta . \cot 3\theta$

LHS

$= \dfrac{\sin 7\theta + \sin 3\theta + 2.\sin 5\theta}{\sin 5\theta + \sin\theta + 2.\sin 3\theta}$

$= \dfrac{2\sin\left[\dfrac{7\theta+3\theta}{2}\right].\cos\left[\dfrac{7\theta-3\theta}{2}\right] + 2.\sin 5\theta}{2\sin\left[\dfrac{5\theta+\theta}{2}\right].\cos\left[\dfrac{5\theta-\theta}{2}\right] + 2.\sin 3\theta}$

$$= \frac{2\sin 5\theta \cdot \cos 2\theta + 2\sin 5\theta}{2\sin 3\theta \cdot \cos 2\theta + 2\sin 3\theta}$$

$$= \frac{2\sin 5\theta \cdot \cancel{(\cos 2\theta + 1)}}{2\sin 3\theta \cdot \cancel{(\cos 2\theta + 1)}}$$

$$= \frac{\sin 5\theta}{\sin 3\theta}$$

$$= \frac{\sin(3\theta + 2\theta)}{\sin 3\theta}$$

$$= \frac{\sin 3\theta \cdot \cos 2\theta}{\sin 3\theta} + \frac{\cos 3\theta \cdot \sin 2\theta}{\sin 3\theta}$$

$$= \cos 2\theta + \sin 2\theta \cdot \cot 3\theta$$

06. $\dfrac{\cos 3A - 2\cos 5A + \cos 7A}{\cos A - 2\cos 3A + \cos 5A} = \cos 2A - \sin 2A \cdot \tan 3A$

LHS

$$= \frac{\cos 7A + \cos 3A - 2\cos 5A}{\cos 5A + \cos A - 2\cos 3A}$$

$$= \frac{2\cos\left(\frac{7A+3A}{2}\right)\cdot\cos\left(\frac{7A-3A}{2}\right) - 2\cos 5A}{2\cos\left(\frac{5A+A}{2}\right)\cdot\cos\left(\frac{5A-A}{2}\right) - 2\cos 3A}$$

$$= \frac{2\cos 5A \cdot \cos 2A - 2\cos 5A}{2\cos 3A \cdot \cos 2A - 2\cos 3A}$$

$$= \frac{2\cos 5A \cdot \cancel{(\cos 2A - 1)}}{2\cos 3A \cdot \cancel{(\cos 2A - 1)}}$$

$$= \frac{\cos 5A}{\cos 3A}$$

$$= \frac{\cos(3A + 2A)}{\cos 3A}$$

$$= \frac{\cos 3A \cdot \cos 2A}{\cos 3A} - \frac{\sin 3A \cdot \sin 2A}{\cos 3A}$$

$$= \cos 2A - \sin 2A \cdot \tan 3A$$

07. $\dfrac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \operatorname{cosec} 2x - \cot 2x$

LHS

$$= \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x}$$

$$= \frac{\sin 5x + \sin x - 2\sin 3x}{\cos 5x - \cos x}$$

$$= \frac{2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) - 2\sin 3x}{-2\sin\left(\frac{5x+x}{2}\right)\cdot\sin\left(\frac{5x-x}{2}\right)}$$

$$= \frac{2\sin 3x \cdot \cos 2x - 2\sin 3x}{-2\sin 3x \cdot \sin 2x}$$

$$= \frac{2\sin 3x\,(\cos 2x - 1)}{-2\sin 3x \cdot \sin 2x}$$

$$= \frac{\cos 2x - 1}{-\sin 2x}$$

$$= \frac{1 - \cos 2x}{\sin 2x}$$

$$= \frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}$$

$$= \operatorname{cosec} 2x - \cot 2x$$

SOLUTION TO Q SET - 3

01. $\dfrac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A - \cos 3A - \cos 4A} = \cot A$

$$\text{LHS} = \frac{\sin 3A + \sin A + \sin 4A + \sin 2A}{\cos A - \cos 3A + \cos 2A - \cos 4A}$$

$$= \frac{2\sin\left(\frac{3A+A}{2}\right).\cos\left(\frac{3A-A}{2}\right) + 2\sin\left(\frac{4A+2A}{2}\right)\cos\left(\frac{4A-2A}{2}\right)}{-2\sin\left(\frac{A+3A}{2}\right).\sin\left(\frac{A-3A}{2}\right) - 2\sin\left(\frac{2A+4A}{2}\right).\sin\left(\frac{2A-4A}{2}\right)}$$

$$= \frac{2.\sin 2A.\cos A + 2.\sin 3A.\cos A}{-2.\sin 2A.\sin(-A) - 2.\sin 3A.\sin(-A)}$$

$$= \frac{2.\sin 2A.\cos A + 2.\sin 3A.\cos A}{2.\sin 2A.\sin A + 2\sin 3A.\sin A}$$

$$= \frac{2.\cos A\ \cancel{(\sin 2A + \sin 3A)}}{2.\sin A\ \cancel{(\sin 2A + \sin 3A)}}$$

$$= \frac{\cos A}{\sin A} = \cot A = \text{RHS}$$

02. $\dfrac{\sin x - \sin 3x + \sin 5x - \sin 7x}{\cos x - \cos 3x - \cos 5x + \cos 7x} = \cot 2x$

$$\text{LHS} = \frac{\sin 5x + \sin x - \sin 7x + \sin 3x}{\cos x - \cos 5x + \cos 7x - \cos 3x}$$

$$= \frac{2\sin\left(\frac{5x+x}{2}\right).\cos\left(\frac{5x-x}{2}\right) - 2\sin\left(\frac{7x+3x}{2}\right)\cos\left(\frac{7x-3x}{2}\right)}{-2\sin\left(\frac{x+5x}{2}\right).\sin\left(\frac{x-5x}{2}\right) - 2\sin\left(\frac{7x+3x}{2}\right)\sin\left(\frac{7x-3x}{2}\right)}$$

$$= \frac{2.\sin 3x.\cos 2x - 2.\sin 5x.\cos 2x}{-2.\sin 3x.\sin(-2x) - 2.\sin 5x.\sin 2x}$$

$$= \frac{2.\sin 3x.\cos 2x - 2.\sin 5x.\cos 2x}{2.\sin 3x.\sin 2x - 2\sin 5x.\sin 2x}$$

$$= \frac{2.\cos 2x\ \cancel{(\sin 3x - \sin 5x)}}{2.\sin 2x\ \cancel{(\sin 3x - \sin 5x)}}$$

$$= \frac{\cos 2x}{\sin 2x} = \cot 2x = \text{RHS}$$

03. $\dfrac{\sin x - \sin 5x + \sin 9x - \sin 13x}{\cos x - \cos 5x - \cos 9x + \cos 13x} = \cot 4x$

$$\text{LHS} = \frac{\sin 9x + \sin x - (\sin 13x + \sin 5x)}{\cos x - \cos 9x + \cos 13x - \cos 5x}$$

$$= \frac{2\sin\left(\frac{9x+x}{2}\right).\cos\left(\frac{9x-x}{2}\right) - 2\sin\left(\frac{13x+5x}{2}\right).\cos\left(\frac{13x-5x}{2}\right)}{-2\sin\left(\frac{x+9x}{2}\right).\sin\left(\frac{x-9x}{2}\right) - 2\sin\left(\frac{13x+5x}{2}\right).\sin\left(\frac{13x-5x}{2}\right)}$$

$$= \frac{2.\sin 5x . \cos 4x - 2.\sin 9x . \cos 4x}{-2.\sin 5x . \sin(-4x) - 2.\sin 9x . \sin 4x}$$

$$= \frac{2.\sin 5x . \cos 4x - 2.\sin 9x . \cos 4x}{2.\sin 5x . \sin 4x - 2\sin 9x . \sin 4x}$$

$$= \frac{2.\cos 4x\,\cancel{(\sin 5x - \sin 9x)}}{2.\sin 4x\,\cancel{(\sin 5x - \sin 9x)}}$$

$$= \frac{\cos 4x}{\sin 4x}$$

$$= \cot 4x = \text{RHS}$$

04. if $\sin 2x + \sin 6x = \cos 2x + \cos 6x$, Show : either $\tan 4x = 1$ OR $\cos 2x = 0$

$$\sin 6x + \sin 2x = \cos 6x + \cos 2x$$

$$2.\sin\left(\frac{6x+2x}{2}\right).\cos\left(\frac{6x-2x}{2}\right) = 2.\cos\left(\frac{6x+2x}{2}\right).\cos\left(\frac{6x-2x}{2}\right)$$

$$2.\sin 4x . \cos 2x = 2.\cos 4x . \cos 2x$$

$$\sin 4x . \cos 2x = \cos 4x . \cos 2x$$

$$\sin 4x . \cos 2x - \cos 4x . \cos 2x = 0$$

$$\cos 2x\,(\sin 4x - \cos 4x) = 0$$

$\cos 2x = 0$ ... PROVED

OR

$\sin 4x - \cos 4x = 0$

$\sin 4x = \cos 4x$

$\dfrac{\sin 4x}{\cos 4x} = 1$

$\tan 4x = 1$ ..... PROVED

05. if $\sin A + \sin 2A + \sin 3A = \cos A + \cos 2A + \cos 3A$,

Show : either $\tan 2A = 1$ OR $\cos A = -1/2$

$$\sin 3A + \sin A + \sin 2A = \cos 3A + \cos A + \cos 2A$$

$$2.\sin\left(\frac{3A + A}{2}\right).\cos\left(\frac{3A - A}{2}\right) + \sin 2A = 2.\cos\left(\frac{3A + A}{2}\right).\cos\left(\frac{3A - A}{2}\right) + \cos 2A$$

$$2.\sin 2A.\cos A + \sin 2A = 2.\cos 2A.\cos A + \cos 2A$$

$$\sin 2A.(2\cos A + 1) = \cos 2A.(2\cos A + 1)$$

$$\sin 2A.(2\cos A + 1) - \cos 2A.(2\cos A + 1) = 0$$

$$(2\cos A + 1)(\sin 2A - \cos 2A) = 0$$

$2\cos A + 1 = 0$ OR $\sin 2A - \cos 2A = 0$

$2,\cos A = -1$ $\qquad$ $\sin 2A = \cos 2A$

$\cos A = -1/2$ $\qquad$ $\dfrac{\sin 2A}{\cos 4A} = 1$

$\tan 2A = 1$ ..... PROVED

06. $\sin 10^\circ + \sin 50^\circ - \sin 80^\circ + \sin 140^\circ = \sqrt{2}.\sin 25^\circ$

LHS =

$\sin 50^\circ + \sin 10^\circ + \sin 140^\circ - \sin 80^\circ$

$$= 2\sin\left(\frac{50 + 10}{2}\right).\cos\left(\frac{50 - 10}{2}\right) + 2\cos\left(\frac{140 + 80}{2}\right).\sin\left(\frac{140 - 80}{2}\right)$$

$$= 2\sin 30.\cos 20 + 2\cos 110.\sin 30$$

$$= 2\sin 30\left(\cos 110 + \cos 20\right)$$

$$= 2.\frac{1}{2}\ 2\cos\left(\frac{110 + 20}{2}\right).\cos\left(\frac{110 - 20}{2}\right)$$

$$= 2\cos 65.\cos 45$$

$$= 2\frac{1}{\sqrt{2}}\cos 65$$

$$= \sqrt{2}\sin 25 = \text{RHS}$$

07. $\cos 40^\circ + \cos 50^\circ + \cos 70^\circ + \cos 80^\circ = \cos 20^\circ + \cos 10^\circ$

LHS =

$\cos 40^\circ + \cos 50^\circ + \cos 70^\circ + \cos 80^\circ$

$= \cos 80^\circ + \cos 40^\circ + \cos 70^\circ + \cos 50^\circ$

$= 2\cos\left(\frac{80+40}{2}\right).\cos\left(\frac{80-40}{2}\right) + 2\cos\left(\frac{70+50}{2}\right).\cos\left(\frac{70-50}{2}\right)$

$= 2\cos 60 . \cos 20 + 2\cos 60 . \cos 10$

$= 2\cos 60 . \left(\cos 20 + \cos 10\right)$

$= 2\ \frac{1}{2}\left(\cos 20 + \cos 10\right)$

$= \cos 20 + \cos 10$

SOLUTION TO Q SET - 4

01. $\dfrac{\sin 3A . \cos 4A - \sin A . \cos 2A}{\sin A . \sin 4A + \cos A . \cos 6A} = \tan 2A$

$$\text{LHS} = \frac{2. \cos 4A . \sin 3A - 2. \cos 2A . \sin A}{2. \sin 4A . \sin A + 2. \cos 6A . \cos A}$$

$$= \frac{\sin(4A+3A) - \sin(4A-3A) - \left(\sin(2A+A) - \sin(2A-A)\right)}{\cos(4A-A) - \cos(4A+A) + \cos(6A+A) + \cos(6A-A)}$$

$$= \frac{\sin 7A - \sin A - \left(\sin 3A - \sin A\right)}{\cos 3A - \cos 5A + \cos 7A + \cos 5A}$$

$$= \frac{\sin 7A - \sin A - \sin 3A + \sin A}{\cos 3A - \cos 5A + \cos 7A + \cos 5A}$$

$$= \frac{\sin 7A - \sin 3A}{\cos 7A + \cos 3A}$$

$$= \frac{2\cos\left(\frac{7A+3A}{2}\right) . \sin\left(\frac{7A-3A}{2}\right)}{2\cos\left(\frac{7A+3A}{2}\right) . \cos\left(\frac{7A-3A}{2}\right)}$$

$$= \frac{2.\cos 5A . \sin 2A}{2.\cos 5A . \cos 2A} = \frac{\sin 2A}{\cos 2A} = \tan 2A$$

02. $\dfrac{\sin 8\theta.\cos\theta - \sin 6\theta.\cos 3\theta}{\cos 2\theta.\cos\theta - \cos 3\theta.\cos 4\theta} = \cot 5\theta$

$$\text{LHS} = \frac{2.\sin 8\theta\ .\cos\theta - 2.\sin 6\theta.\cos 3\theta}{2.\cos 2\theta.\cos\theta - 2.\cos 4\theta.\cos 3\theta}$$

$$= \frac{\sin(8\theta+\theta) + \sin(8\theta-\theta) - \left[\sin(6\theta+3\theta) + \sin(6\theta-3\theta)\right]}{\cos(2\theta+\theta) + \cos(2\theta-\theta) - \left[\cos(4\theta+3\theta) + \cos(4\theta-3\theta)\right]}$$

$$= \frac{\sin 9\theta + \sin 7\theta - \left[\sin 9\theta + \sin 3\theta\right]}{\cos 3\theta + \cos\theta - \left[\cos 7\theta + \cos\theta\right]}$$

$$= \frac{\sin 9\theta + \sin 7\theta - \sin 9\theta - \sin 3\theta}{\cos 3\theta + \cos\theta - \cos 7\theta - \cos\theta}$$

$$= \frac{\sin 7\theta - \sin 3\theta}{\cos 3\theta - \cos 7\theta}$$

$$= \frac{2\cos\left[\frac{7\theta+3\theta}{2}\right].\sin\left[\frac{7\theta-3\theta}{2}\right]}{-2\sin\left[\frac{3\theta+7\theta}{2}\right].\sin\left[\frac{3\theta-7\theta}{2}\right]}$$

$$= \frac{2.\cos 5\theta\ .\ \sin 2\theta}{-\ 2.\sin 5\theta\ .\ \sin(-2\theta)}$$

$$= \frac{2.\cos 5\theta\ .\ \sin 2\theta}{2.\ \sin 5\theta\ .\ \sin 2\theta} = \frac{\cos 5\theta}{\sin 5\theta} = \cot 5\theta$$

03. $\dfrac{\sin 3\theta.\cos 5\theta - \sin\theta.\ \cos 7\theta}{\sin\theta.\ \sin 7\theta - \cos 3\theta.\cos 5\theta} = \tan 2\theta$

$$\text{LHS} = \frac{2.\cos 5\theta.\sin 3\theta - 2.\cos 7\theta.\ \sin\theta}{2.\sin 7\theta\ .\ \sin\theta + 2.\cos 5\theta.\cos 3\theta}$$

$$= \frac{\sin(5\theta+3\theta) - \sin(5\theta-3\theta) - \left[\sin(7\theta+\theta) - \sin(7\theta-\theta)\right]}{\cos(7\theta-\theta) - \cos(7\theta+\theta) + \cos(5\theta+3\theta) + \cos(5\theta-3\theta)}$$

$$= \frac{\sin 8\theta - \sin 2\theta - \left[\sin 8\theta - \sin 6\theta\right]}{\cos 6\theta - \cos 8\theta + \cos 8\theta + \cos 2\theta}$$

$$= \frac{\sin 8\theta - \sin 2\theta - \sin 8\theta + \sin 6\theta}{\cos 6\theta - \cos 8\theta + \cos 8\theta + \cos 2\theta}$$

$$= \frac{\sin 6\theta - \sin 2\theta}{\cos 6\theta + \cos 2\theta}$$

$$= \frac{2\cos\left(\frac{6\theta + 2\theta}{2}\right).\sin\left(\frac{6\theta - 2\theta}{2}\right)}{2\cos\left(\frac{6\theta + 2\theta}{2}\right).\cos\left(\frac{6\theta - 2\theta}{2}\right)}$$

$$= \frac{2.\cos 4\theta . \sin 2\theta}{2.\cos 4\theta . \cos 2\theta}$$

$$= \frac{\sin 2\theta}{\cos 2\theta}$$

$$= \cot 2\theta$$

04. $$\frac{\cos 3A . \sin 9A - \sin A . \cos 5A}{\cos A . \cos 5A - \sin 3A . \sin 9A} = \tan 8A$$

LHS $$= \frac{2. \sin 9A . \cos 3A - 2. \cos 5A . \sin A}{2. \cos 5A . \cos A - 2. \sin 9A . \sin 3A}$$

$$= \frac{\sin(9A + 3A) + \sin(9A - 3A) - [\sin(5A + A) - \sin(5A - A)]}{\cos(5A + A) + \cos(5A - A) - [\cos(9A - 3A) - \cos(9A + 3A)]}$$

$$= \frac{\sin 12A + \sin 6A - [\sin 6A - \sin 4A]}{\cos 6A + \cos 4A - [\cos 6A - \cos 12A]}$$

$$= \frac{\sin 12A + \sin 6A - \sin 6A + \sin 4A}{\cos 6A + \cos 4A - \cos 6A + \cos 12A}$$

$$= \frac{\sin 12A + \sin 4A}{\cos 12A + \cos 4A}$$

$$= \frac{2\sin\left(\frac{12A + 4A}{2}\right).\cos\left(\frac{12A - 4A}{2}\right)}{2\cos\left(\frac{12A + 4A}{2}\right).\cos\left(\frac{12A - 4A}{2}\right)}$$

$$= \frac{2.\sin 8A . \cos 4A}{2.\cos 8A . \cos 4A}$$

$$= \frac{\sin 8A}{\cos 8A} = \tan 8A = \text{RHS}$$

05. $\dfrac{\cos 7A}{\cos A} = 2.\cos 6A - 2.\cos 4A + 2.\cos 2A - 1$

WE PROVE

$\cos 7A = \cos A\,(2.\cos 6A - 2.\cos 4A + 2.\cos 2A - 1)$

RHS $= \cos A\,(2.\cos 6A - 2.\cos 4A + 2.\cos 2A - 1)$

$= 2.\cos 6A.\cos A - 2.\cos 4A.\cos A + 2.\cos 2A.\cos A - \cos A$

$= \cos(6A + A) + \cos(6A - A) - \left[\cos(4A + A) + \cos(4A - A)\right] + \cos(2A + A) + \cos(2A - A) - \cos A$

$= \cos 7A + \cos 5A - \cos 5A - \cos 3A + \cos 3A + \cos A - \cos A$

$= \cos 7A$

06. $\sin\left(\dfrac{3A}{2}\right).\sin\left(\dfrac{11A}{2}\right) + \sin\left(\dfrac{A}{2}\right).\sin\left(\dfrac{7A}{2}\right) = \sin 2A . \sin 5A$

LHS $= \dfrac{1}{2}\ 2\ \sin\left(\dfrac{11A}{2}\right).\sin\left(\dfrac{3A}{2}\right) + \dfrac{1}{2}\ 2\ \sin\left(\dfrac{7A}{2}\right) . \sin\left(\dfrac{A}{2}\right)$

$= \dfrac{1}{2}\left\{\cos\left(\dfrac{11A}{2} - \dfrac{3A}{2}\right) - \cos\left(\dfrac{11A}{2} + \dfrac{3A}{2}\right)\right\} + \dfrac{1}{2}\left\{\cos\left(\dfrac{7A}{2} - \dfrac{A}{2}\right) - \cos\left(\dfrac{7A}{2} + \dfrac{A}{2}\right)\right\}$

$= \dfrac{1}{2}\left[\cos 4A - \cos 7A\right] + \dfrac{1}{2}\left[\cos 3A - \cos 4A\right]$

$= \dfrac{1}{2}\left[\cos 4A - \cos 7A + \cos 3A - \cos 4A\right]$

$= \dfrac{1}{2}\left[\cos 3A - \cos 7A\right]$

$= \dfrac{1}{2}\left\{-\ 2\sin\left(\dfrac{3A + 7A}{2}\right) . \sin\left(\dfrac{3A - 7A}{2}\right)\right\}$

$= \dfrac{1}{2}\left[-\ 2\sin 5A . \sin(-2A)\right]$

$= \dfrac{1}{2}\ 2\sin 5A . \sin 2A$

$= \sin 5A . \sin 2A$

SOLUTION TO Q SET - 5

01. $\cos 20^\circ . \cos 100^\circ + \cos 100^\circ . \cos 140^\circ - \cos 140^\circ . \cos 200^\circ = -3/4$

WE PROVE :

$2.\cos 20^\circ . \cos 100^\circ + 2.\cos 100^\circ . \cos 140^\circ - 2.\cos 140^\circ . \cos 200^\circ = -3/2$

LHS $= 2.\cos 100^\circ . \cos 20^\circ + 2.\cos 140^\circ . \cos 100^\circ - 2.\cos 200^\circ . \cos 140^\circ$

$= \cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \left(\cos 340^\circ + \cos 60^\circ\right)$

$= \cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \cos 340^\circ - \cos 60^\circ$

Now : $\cos 120 = \cos(180 - 60) = -\cos 60 = -1/2$
(II QUAD)

$\cos 240 = \cos(180 + 60) = -\cos 60 = -1/2$
(III QUAD)

$\cos 340 = \cos(360 - 20) = +\cos 20$
(IV QUAD)

BACK IN SUM

$= -1/2 + \cos 80^\circ - 1/2 + \cos 40^\circ - \cos 20^\circ - 1/2$

$= -3/2 + \cos 80^\circ + \cos 40^\circ - \cos 20^\circ$

$= -3/2 + 2\cos\left(\frac{80+40}{2}\right)^\circ . \cos\left(\frac{80-40}{2}\right)^\circ - \cos 20^\circ$

$= -3/2 + 2.\cos 60^\circ \cos 20^\circ - \cos 20^\circ$

$= -3/2 + 2.(1/2)\cos 20^\circ - \cos 20^\circ$

$= -3/2 + \cos 20^\circ - \cos 20^\circ$

$= -3/2$

02. $\cos 55^\circ . \cos 65^\circ + \cos 65^\circ . \cos 175^\circ + \cos 175^\circ . \cos 55^\circ = -3/4$

WE PROVE :

$2.\cos 55^\circ . \cos 65^\circ + 2.\cos 65^\circ . \cos 175^\circ + 2.\cos 175^\circ . \cos 55^\circ = -3/2$

LHS $= 2.\cos 65^\circ . \cos 55^\circ + 2.\cos 175^\circ . \cos 65^\circ + 2.\cos 175^\circ . \cos 55^\circ$

$= \cos 120^\circ + \cos 10^\circ + \cos 240^\circ + \cos 110^\circ + \cos 230^\circ + \cos 120^\circ$

Now : $\cos 120 = \cos(180 - 60) = -\cos 60 = -1/2$
(II QUAD)

$\cos 240 = \cos(180 + 60) = -\cos 60 = -1/2$
(III QUAD)

$\cos 230 = \cos(180 + 50) = -\cos 50$
(III QUAD)

BACK IN SUM

$= -1/2 + \cos 10^\circ - 1/2 + \cos 110^\circ - \cos 50^\circ - 1/2$

$= -3/2 + \cos 110^\circ + \cos 10^\circ - \cos 50^\circ$

$= -3/2 + 2\cos\left(\dfrac{110+10}{2}\right)^\circ . \cos\left(\dfrac{110-10}{2}\right)^\circ - \cos 50^\circ$

$= -3/2 + 2.\cos 60^\circ \cos 50^\circ - \cos 50^\circ$

$= -3/2 + 2.(1/2)\cos 50^\circ - \cos 50^\circ$

$= -3/2 + \cos 50^\circ - \cos 50^\circ$

$= -3/2$

= RHS

03. $\sin 25^\circ . \sin 35^\circ - \sin 25^\circ . \sin 85^\circ - \sin 35^\circ . \sin 85^\circ = - 3/4$

WE PROVE :

$2.\sin 25^\circ . \sin 35^\circ - 2.\sin 25^\circ . \sin 85^\circ - 2.\sin 35^\circ . \sin 85^\circ = - 3/2$

LHS $= 2.\sin 35^\circ . \sin 25^\circ - 2.\sin 85^\circ . \sin 25^\circ - 2.\sin 85^\circ . \sin 35^\circ$

$= \cos 10^\circ - \cos 60^\circ - (\cos 60^\circ - \cos 110^\circ) - (\cos 50^\circ - \cos 120^\circ)$

$= \cos 10^\circ - \cos 60^\circ - \cos 60^\circ + \cos 110^\circ - \cos 50^\circ + \cos 120^\circ$

Now : $\cos 120 = \cos(180 - 60) = - \cos 60 = - 1/2$

$= \cos 10^\circ - 1/2 - 1/2 + \cos 110^\circ - \cos 50^\circ - 1/2$

$= -3/2 + \cos 110^\circ + \cos 10^\circ - \cos 50^\circ$

$= -3/2 + 2\cos\left(\dfrac{110+10}{2}\right)^\circ . \cos\left(\dfrac{110-10}{2}\right)^\circ - \cos 50^\circ$

$= -3/2 + 2.\cos 60^\circ \cos 50^\circ - \cos 50^\circ$

$= -3/2 + 2.(1/2)\cos 50^\circ - \cos 50^\circ$

$= -3/2 + \cos 50^\circ - \cos 50^\circ$

$= -3/2$

= RHS

04. $\cos^2 x + \cos^2(x + 120) + \cos^2(x - 120) = 3/2$

We Prove : $2\cos^2 x + 2\cos^2(x + 120) + 2\cos^2(x - 120) = 3$

$1 + \cos 2\theta = 2\cos^2\theta$

$$\begin{aligned}
\text{LHS} &= 2\cos^2 x + 2\cos^2(x + 120) + 2\cos^2(x - 120) \\
&= 1 + \cos 2x + 1 + \cos(2x + 240) + 1 + \cos(2x - 240) \\
&= 3 + \cos 2x + \cos(2x + 240) + \cos(2x - 240) \\
&= 3 + \cos 2x + 2 \cdot \cos\left(\frac{2x + 240 + 2x - 240}{2}\right) \cos\left(\frac{2x + 240 - 2x + 240}{2}\right) \\
&= 3 + \cos 2x + 2 \cdot \cos 2x \cdot \cos 240 \\
&= 3 + \cos 2x + 2 \cdot \cos 2x \cdot \frac{-1}{2} \\
&= 3 + \cos 2x - \cos 2x \\
&= 3
\end{aligned}$$

04. $\cos^2 x + \cos^2(x + 300) + \cos^2(x - 300) = 3/2$

We Prove : $2\cos^2 x + 2\cos^2(x + 300) + 2\cos^2(x - 300) = 3$

$1 + \cos 2\theta = 2\cos^2\theta$

$$\begin{aligned}
\text{LHS} &= 2\cos^2 x + 2\cos^2(x + 300) + 2\cos^2(x - 300) \\
&= 1 + \cos 2x + 1 + \cos(2x + 600) + 1 + \cos(2x - 600) \\
&= 3 + \cos 2x + \cos(2x + 600) + \cos(2x - 600) \\
&= 3 + \cos 2x + 2 \cdot \cos\left(\frac{2x + 600 + 2x - 600}{3}\right) \cos\left(\frac{2x + 600 - 2x + 600}{2}\right) \\
&= 3 + \cos 2x + 2 \cdot \cos 2x \cdot \cos 600 \\
&= 3 + \cos 2x + 2 \cdot \cos 2x \cdot \left(\frac{-1}{2}\right) \\
&= 3 + \cos 2x - \cos 2x \\
&= 3
\end{aligned}$$

$$\begin{aligned}
\cos 600 &= \cos(540 + 60) \\
&= \cos(3 \times 180 + 60) \\
&= -\cos 60 \\
&= -{}^{1}/_{2}
\end{aligned}$$

05. $\sin^2\theta + \sin^2(120 + \theta) + \sin^2(120 - \theta) = 3/2$

We Prove : $2 . \sin^2\theta + 2 . \sin^2(120 + \theta) + 2 . \sin^2(120 - \theta) = 3$

$1 - \cos 2\theta = 2\sin^2\theta$

$$\begin{aligned}
\text{LHS} &= 2. \sin^2\theta + 2 . \sin^2(120 + \theta) + 2 . \sin^2(120 - \theta) \\
&= 1 - \cos 2\theta + 1 - \cos (240 + 2\theta) + 1 - \cos (240 - 2\theta) \\
&= 3 - \cos 2\theta - \cos (240 + 2\theta) - \cos (240 - 2\theta) \\
&= 3 - \cos 2\theta - \left[\cos (240 + 2\theta) + \cos (240 - 2\theta)\right] \\
&= 3 - \cos 2\theta - 2 . \cos \left(\frac{240 + 2\theta + 240 - 2\theta}{4}\right) \cos \left(\frac{240 + 2\theta - 240 + 2\theta}{2}\right) \\
&= 3 - \cos 2\theta - 2 . \cos 240 . \cos 2\theta \\
&= 3 - \cos 2\theta - 2 . \frac{-1}{2} . \cos 2\theta \\
&= 3 - \cos 2\theta + \cos 2\theta \\
&= 3
\end{aligned}$$